B

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER M.TECH. DEGREE EXAMINATION, DECEMBER 2017

CIVIL ENGINEERING

10CE6103: THEORY OF ELASTICITY

Max. Marks : 60 Duration: 3 Hrs.

Part A (Modules I - II)

*(Answer any **two** questions : 2 × 9 = 18 Marks)*

1. a) Derive the differential equations of equilibrium. (5 marks)

   b) The state of stress at a point in a stressed body is given by the following stress components: σ_x =75 MPa, σ_y = 60 MPa, σ_z = 50 MPa, τ_{xy} = 25 MPa, τ_{yz} = —25 MPa and τ_{xz} = 30 MPa. Determine the normal and shear stresses on a plane having directions cosines of its outer normal as cos (N, x) =12/25, cos (N, y) = 15/25 and cos (N, z) =16/25. (4 marks)

2. a) Derive the compatibility conditions. (6 marks)

   b) The strain components at a point are given by ε_x =0. 1, ε_y = — 0.05, ε_z = 0.05, γ_{xy} = 0.3, γ_{yz} =0.1, γ_{xz} = — 0.08. Determine the principal strains. (3 marks)

3. a) If the stress field is given by

$$\sigma_x = 3xy^2z + 2x, \qquad \tau_{xy} = 0$$
$$\sigma_y = 5xyz + 3y \qquad \tau_{yz} = \tau_{xz} = 3xy^2z + 2xy$$
$$\sigma_z = x^2y + y^2z$$

   Determine whether these components of stress satisfy the equilibrium equations or not as the point (1, -1, 2). If not, determine the suitable body force required at this point so that these stress components are under equilibrium. (3 marks)

b) The state of stress at a point in a stressed body is given by the following stress components: σ_x =20 MPa, σ_y = ─40 MPa, σ_z = 80 MPa, τ_{xy} = 40 MPa, τ_{yz} = -60 MPa and τ_{xz} = 20 MPa. Determine the principal stresses, deviatoric and spherical stress tensors. (6 marks)

Part B (Modules III - IV)

*(Answer any **two** questions : 2 × 9 = 18 Marks)*

4. a) State and explain the St. Venant's principle. (3 marks)

b) The displacement at a point (*x, y*) are as given below

$$u = 5x^4 + 3x^2y^2 + x + y$$
$$v = y^3 + 2xy + 4$$

Compute the values of normal and shearing strains at a point (3, -2) and verify whether compatibility exists or not? (5 marks)

c) At a point in a stressed material, the state of strain is determined as follows:

$$\varepsilon_x = 0.001, \quad \varepsilon_y = -0.003, \quad \varepsilon_z = \gamma_{xy} = 0, \quad \gamma_{xz} = -0.004, \quad \gamma_{yz} = 0.001$$

Calculate the volumetric strain and the Lame's constants if E = 210 kN / mm^2 and Poisson's ratio is 0.3. (2 marks)

5. a) State and explain the Generalised Hooke's law and deduce the simplifications possible for orthotropic, transversely isotropic and isotropic media. (5 marks)

b) Discuss a problem of plane stress that can be solved using a 4^{th} degree polynomial. (4 marks)

6. Determine the stresses and displacements developed in a cantilever of span L, depth 2h and unit width subjected to a point load at the free end using Airy's stress function approach. (9 marks)

Part C (Modules V & VI)

(Answer any two questions : 2 × 12 = 24 Marks)

7. a) Is the following expression, a stress function?

$$\phi = -\left(\frac{P}{\pi}\right) r\theta \sin\theta$$

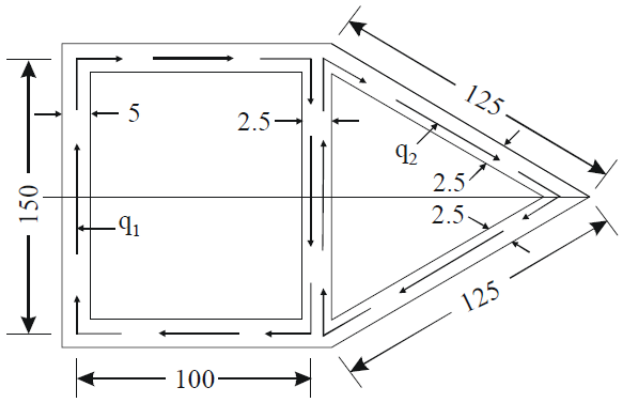
If so, find the corresponding stresses. (4 marks)

b) Derive the expression for stresses in a shaft of equilateral triangular cross-section under torsion and plot the variation of stress. (8 marks)

8. a) Derive the equations of equilibrium in polar coordinates. (8 marks)

b) The internal and external diameters of a thick hollow cylinder are 80 mm and 120 mm respectively. It is subjected to an external pressure of 40 MN/m^2, when the internal pressure is 120 MN/m^2. Calculate the circumferential stresses at the external and internal surfaces and determine the radial and circumferential stresses at the mean radius. (4 marks)

9. A two-cell tube as shown in the figure below is subjected to a torque of 10 kN-m. Determine the shear stress in each part and the angle of twist per metre length. Assume G = 83 kN/mm^2.



All dimensions in mm

(12 marks)
